

Electric Loads Modeling in the Presence of Harmonics Based on Z-Transform

Ahmad M. Alkandari, Jamal Y. Madouh, Bader A. Alkandari, Bader Alfouzan
and Soliman A. Soliman

We present in this paper an approach based on Z-transform to model single phase electric loads in the presence of harmonics. The impedance or admittance of the load is expressed as a linear autoregressive exogenous input (ARX). The parameters of this series are identified using the least error squares (LES) algorithm. This algorithm uses the digitized samples of the load-input voltage and the resulting output current. Having identified the ARX parameters model, the impedance, or admittance of the equivalent circuit of the load can be written in the time domain. Using the well-known definition of the per unit Z, the admittance of the load can be expressed in the frequency domain. The proposed algorithm can easily be used to model the impedance or admittance of the electric distribution network on the harmonic polluted environment, as well as the transmission network, for load flow, stability analysis and harmonics filter design. The results of simulated and actual recorded data are reported in this paper.

Keywords: Z transform, electric load modeling, harmonics, LES estimation

1. Introduction

The widespread use of power electronics in transmission and distribution of electric systems as well as nonlinear loads such as electric drives, arc furnace, all kind of lighting devices produce harmonics in the power networks and an accurate model of power system components is essential. Modeling of electric loads in the presence of harmonics pays the attention of researchers and power engineering utility companies to predict the load effects during power system transients as well as steady state power flow analysis. Many techniques are developed in the last two decades, and are grouped into two categories; the time domain approach and frequency domain approach. Some of these techniques have been applied to model the nonlinear single-phase and three phase loads.

References [1] and [10] present a technique for modeling linear or nonlinear loads in the presence or absence of harmonics in the time-domain. The principle of energy stored in the load elements is applied. The residual currents as well as the mismatch current for different harmonic frequencies in the voltage or current waveforms are calculated. The accuracy of the model produced in this reference is tested using the HARMFLO program that calculates the power flow at different harmonic orders.

Ahmad M. Alkandari, Jamal Y. Madouh, Bader A. Alkandari and Bader Alfouzan, College of Technological Studies, Electrical Engineering Technology Department, Shuweikh, Kuwait, E-mail: alkandari1@yahoo.com
Soliman A. Soliman, Misr University for Science and Technology, Electrical Power and Machines Department, Giza, Egypt, E-mail: sasoliman@must.edu.eg

A method for modeling distribution load impedances as unbalanced three phase impedance matrices for detailed harmonic studies is presented in Reference [2]. The linear time series is used, the parameters of this series are calculated using the least error squares algorithm and they are considered constant during the horizon time

Reference [3] developed a computer program that simulates the effect of harmonics introduced at distribution level by nonlinear loads. A table of harmonics (amplitude and phase) represents each load device, of the current when energized by a sinusoidal voltage. The purpose of the program is to allow load-flow studies to include effects of harmonics particularly on the customer supply voltage. The three-phase load in the program is assumed to be balanced.

Reference [4] describes an improved model of rotating loads in complex load areas, for harmonic penetration analyses applied to distribution systems. The analysis is performed in the frequency domain. The proposed model can be used for simulating large complex load areas within harmonic studies for which the injection current method has been found to be still sufficiently accurate.

Reference [5] presents two time domain techniques for load modeling in the presence of harmonics. In the first techniques, the harmonic contents of the load voltage and current waveforms are assumed to be known in advance. While, the second technique uses directly samples of load voltage and current waveforms. The least error squares parameter estimation algorithm is used to estimate the load resistance, inductance and capacitance. Residual current, results from incomplete extraction of the parameters for all significant signal frequencies, is also estimated. Harmonics that are not common between load voltage and current waveforms are also identified.

The cross frequency admittance matrix for presenting non-linear loads is presented in Reference [6]. The model is applicable to passive and stationary electrical loads providing that the fundamental frequency voltage is constant. This approach linearizes the load behavior around its working point, to define a “crossed-frequency” harmonic admittance matrix that correlate harmonic currents and voltages of different orders.

Nonlinear electric loads result in harmonics in voltages and current waveforms. This has prompted the need for more accurate models for these loads, in order to allow accurate prediction and mitigation of harmonic distortion. Reference [7] presents a general methodology for modeling linear or nonlinear loads, in the frequency domain, in the presence of harmonics. The proposed method is based on a least errors squares algorithm and requires advances knowledge of the harmonic contents of load voltage and current waveforms. It has been shown that the orders of the load transfer admittance, as well as, number of parameters to be identified depend on the number of common harmonics between the voltage and current waveforms.

Effects of load modeling for under voltage load shedding are presented in Reference [8]. A dynamic and static load models are presented for showing how much load be to shed and what influence different load models have on the simulation result as well as on the analysis.

The analysis of the load harmonic model under unbalanced condition is presented in Reference [10]. Different load models at the frequency domain have been derived for linear and nonlinear loads. The models of linear and non-linear loads are incorporated to the three-phase harmonic load flow formulation.

Reference [11] presents a fast and efficient on-line identification of non-linear loads in power systems. The proposed technique is an application of a discrete time-dynamic filter based on stochastic estimation theory which is suitable for

estimating parameters on-line. The algorithm uses sets of digital samples of the distorted voltage and current waveforms of the non-linear load to estimate the harmonic contents of these two signals. The non-linear load admittance is then calculated from these contents.

This paper presents an approach based on the Z-transform to model single phase electric loads in the presence of harmonics. The impedance or admittance of the load is expressed as linear autoregressive exogenous input (ARX). The parameters of this series are identified using the least error squares (LES) estimation algorithm, where the digitized samples of the load input voltage and the resulting output current are used. Having identified the ARX parameters model, the impedance, or admittance of the equivalent circuit of the load can be expressed in time domain. Using the well-known definition of the per unit Z, the admittance of the load can be written in the frequency domain. The proposed algorithm can easily be used to model the impedance or admittance of the electric distribution network on the harmonic polluted environment, as well as the transmission network, for load flow, stability analysis and harmonics filter design. Results of simulated and actual recorded data for modeling single-phase loads in the presence of harmonics are reported in this paper

2. MODELING SINGLE PHASE LOADS

In this model the load current is presented as a series function of the voltage and current at time t as:

$$i(t) = a_0 v(t) + a_1 v(t - 1) + a_2 v(t - 2) + \dots + a_m v(t - m) \quad (1)$$

Where a_0, a_1, a_2, a_m are the load parameters to be estimated. Using the definition of the per unit operator $Z^{-1} v(t) = v(t-1)$, and $Z^{-m} v(t) = v(t-m)$, where $Z = \exp(-j2\pi f/f_s)$, then equation (1) can be written as:

$$i(z) = (a_0 + a_1 Z^{-1} + a_2 Z^{-2} + \dots + a_m Z^{-m}) v(z) \quad (2)$$

The load admittance $Y(z)$ is defined as:

$$Y(z) = \frac{i(z)}{v(z)} = a_0 + a_1 Z^{-1} + a_2 Z^{-2} + \dots + a_m Z^{-m} \quad (3)$$

The parameters of the admittance in equation (3) can be identified using the least error squares algorithm. Equation (1) can be written in vector form as

$$\mathbf{I}(t) = \mathbf{A}(t) \mathbf{X} + \mathbf{g}(t) \quad (4)$$

Where $\mathbf{I}(t)$ is an $m \times 1$ measurement vector of current samples, $\mathbf{A}(t)$ is an $m \times n$ matrix of voltage samples of the recent sample and the previous samples, \mathbf{X} is $n \times 1$ parameters vector of the load model to be identified, and $\mathbf{g}(t)$ errors vector to be minimized. The solution to equation (4) based on the least errors square algorithm is

$$\mathbf{X} = [\mathbf{A}^T(t) \mathbf{A}(t)]^{-1} \mathbf{A}^T(t) \mathbf{I}(t) \quad (5)$$

Having identified the load model parameters from equation (5), then the load admittance of equation (3) can be calculated at

any harmonic frequency f , just by replacing Z by $Z = \exp(-j2\pi \frac{f}{f_s})$, with f_s is the sampling frequency used to obtain the

earlier required samples in equation (1).

3. RESULTS FOR MODEL I [8]

Two examples are offered in this section. The first one is a simulated example. The load voltage and current waveforms for this example are shown in Figure 1. The harmonics content for each waveform is given by

$$v(t) = \sqrt{2}[\cos \omega t + 0.6 \cos 3 \omega t], \text{ and}$$

$$i(t) = \sqrt{2}[\cos(\omega t - 30^\circ) + 0.25 \cos(3 \omega t + 60^\circ)]$$

Note that, the proposed algorithm uses directly the samples of the voltage and current, and no need to identify the harmonics content of each waveform. The estimated parameters of the transfer function of Equation (3) are:

$$a_0 = 0.633, \quad a_1 = 3.731 \times 10^{-3}, \quad a_2 = 0.336, \quad a_3 = -0.2439, \quad a_4 = 0.04803, \quad a_5 = 0.09878, \\ a_6 = -0.0167, \quad a_7 = -0.038424, .$$

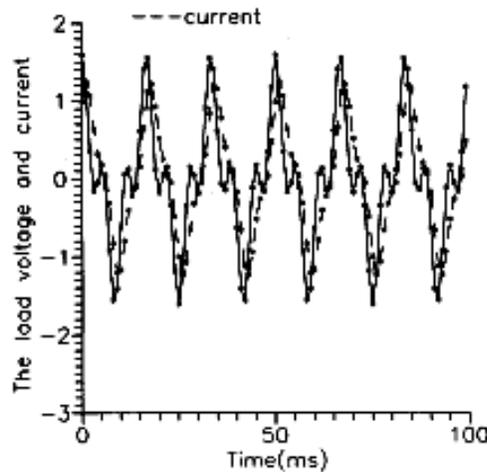


Figure 1. The voltage and current waveform

The load conductance and susceptance are given in Figures (2) and (3). In Figure 2, the load conductance is approximated by a polynomial of order 10 as:

$$G(f) = -0.92132 + 0.048932f - 0.000428f^2 + 1.313 \times 10^{-6}f^3 - 3.3175 \times 10^{-10}f^4 - 6.87 \times 10^{-12}f^5 \\ + 1.7337 \times 10^{-14}f^6 - 2.01 \times 10^{-17}f^7 + 1.259 \times 10^{-20}f^8 - 4.1338 \times 10^{-24}f^9 + 5.581 \times 10^{-28}f^{10}$$

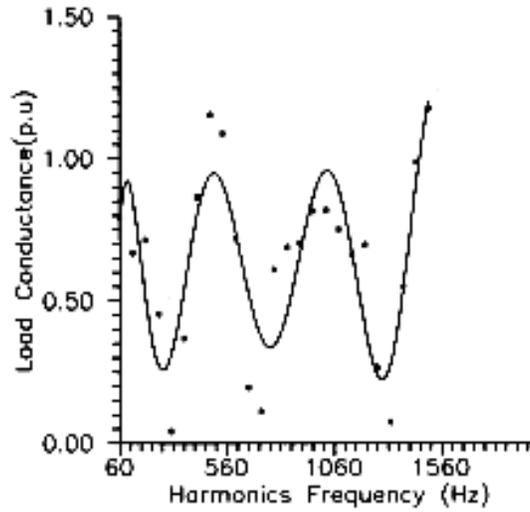


Figure 2 Variation of load conductance with frequency

*** Estimated _____ best fit

Where f is the frequency harmonic in Hz. It can be seen that the load has a negative dc resistance, which means that the load can supply power to the circuit at $f=0.0$, which never happens, for power system network. However, if the phase angle between a harmonic voltage and a harmonic current of the same frequency is greater than 90° , then the power resulting from this voltage and current will be negative, i.e. they deliver power to the network.

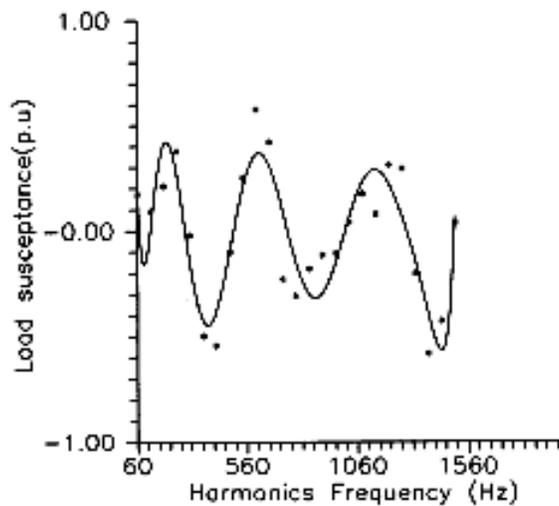


Figure 3 Variation of load susceptance with frequency

*** Estimated _____ best fit

While the load susceptance is approximated by a polynomial of order 10 as:

$$B(f) = 6.14275 - 0.21f + 0.00258f^2 - 1.56710 \times 10^{-5}f^3 + 5.333 \times 10^{-8}f^4 - 1.1 \times 10^{-10}f^5 + 1.4273 \times 10^{-13}f^6 - 1.1736 \times 10^{-16}f^7 + 5.937 \times 10^{-20}f^8 - 1.687 \times 10^{-23}f^9 + 2.0628 \times 10^{-27}f^{10}$$

Examining these two figures reveals:

- The load conductance and susceptance depend on the harmonic frequencies.
- The load conductance in the range of 60 Hz and up is always positive, i.e. in this range the load absorbed power from the network.
- The load changes its mode of operation from inductive load to capacitive load according to the harmonic frequency under study.
- There are many frequencies at which the load is working as a pure resistance (resonance frequencies), these can easily be obtained by taking the frequency at which the load susceptance is zero.

The second example is an actual recorded data for a large 1250 HP induction motor connected to a 44 kV power system bus [10]. A 6-pulse converter type drives the motor, which is the source of harmonics. The load voltage and current waveforms are given in Figure (4). The harmonics contents of these waveforms are::

$$v(t) = 0.2822 \cos(\omega t - 28.6^\circ) + 0.0016 \cos(3\omega t - 22.5^\circ) + 0.01152 \cos(5\omega t - 162.4^\circ) + 0.01274 \cos(7\omega t + 145.2^\circ)$$

$$i(t) = -\cos(\omega t + 100^\circ) + 0.027 \cos(3\omega t) + 0.151 \cos(5\omega t) + 0.1335 \cos(7\omega t)$$

The estimated parameters for the load admittance transfer function are:

$$a_0 = 1.477, \quad a_1 = -0.3377, \quad a_2 = 1.61, \quad a_3 = -0.835, \quad a_4 = 0.3463, \quad a_5 = -0.219, \quad a_6 = 0.0784, \quad a_7 = -0.0754.$$

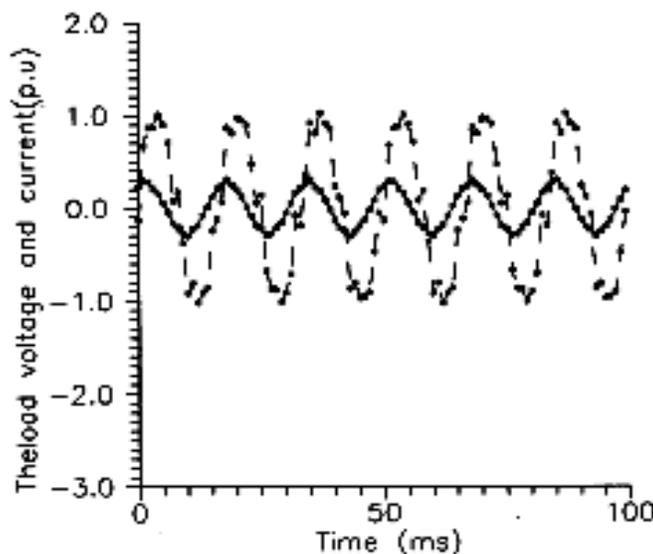


Figure 4 Voltage and current waveforms

Figure (5), and (6) give the estimated load conductance and susceptance. The load conductance is approximated by a polynomial of order 8 at a harmonic frequency as:

$$G(f) = -5.1886 + 0.236426f - 0.0026f^2 + 1.194 \times 10^{-5} f^3 - 2.872 \times 10^{-8} f^4 + 3.84 \times 10^{-11} f^5 - 2.8 \times 10^{-14} f^6 + 1.134 \times 10^{-17} f^7 - 1.822 \times 10^{-21} f^8$$

While the load susceptance is approximated by a polynomial of order 10 as:

$$B(f) = 21.574 - 0.76f + 0.00974f^2 - 6.09 \times 10^{-5} f^3 + 2.118 \times 10^{-7} f^4 - 4.4354 \times 10^{-10} f^5 + 5.823 \times 10^{-13} f^6 - 4.831 \times 10^{-16} f^7 + 2.461 \times 10^{-19} f^8 - 7.034 \times 10^{-23} f^9 + 8.644 \times 10^{-27} f^{10}$$

Examining these two figures reveals the following:

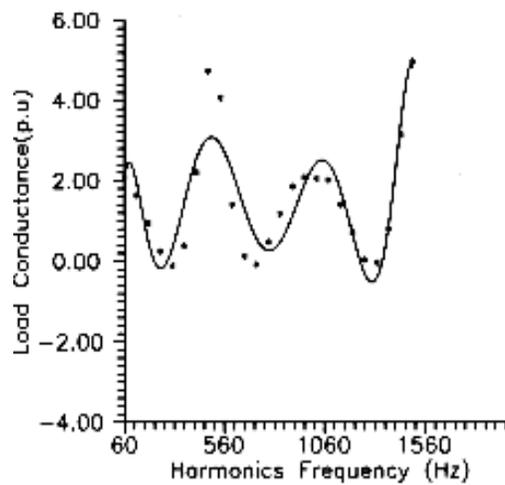


Figure 5 Variation of load conductance with frequency

*** Estimated _____ best fit

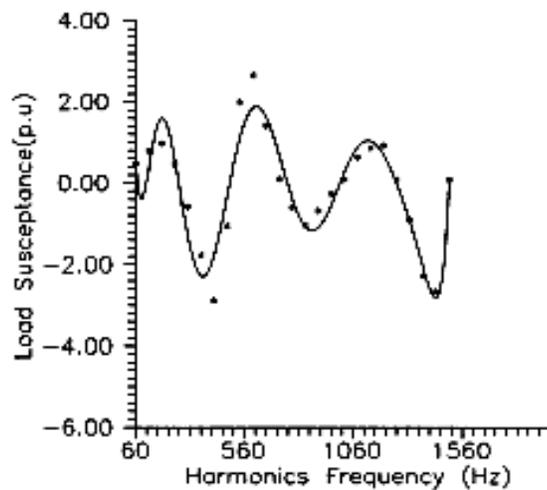


Figure 6 Variation of load susceptance with frequency

*** Estimated _____ best fit

1. The motor has always a positive conductance for all the harmonic frequency ranges.
2. There are many frequencies at which the load acts as a pure resistance (resonance frequencies)
3. If one examines the $B(f)$ curve, it can be easily noticed that there are harmonic frequencies at which the load behaves as an inductive load ($-ve B(f)$) while another frequency it behaves as a capacitive load ($+ve B(f)$)

4. CONCLUSIONS

A model for harmonic modeling of a single-phase load is presented in this paper, in the time domain. The model is a polynomial of the Z- transform, where the parameters of this model are identified using the available samples of the current and the voltage waveform. In the model the recent sample of the current is used, while the previous and recent samples of the voltage are used. The least errors square algorithm is used to estimate the parameters of the loads. A set of polynomial equations for the conductance and the susceptance of the model are derived. It has been noticed that the proposed model is suitable for the harmonic analysis of the system that contains such nonlinear loads. The model can be used in harmonic power flow analysis, and system stability analysis.

The developed model in this paper is for single-phase loads. It is worthwhile to develop a suitable model for a three phase unbalanced load, which is our current research.

5. REFERENCES

1. S. Varadan, and E.B. Makram, "Generalized Load Modeling in the Presence of Harmonics and Distortion", Twenty-Fifth Southeastern Symposium on System Theory, 1993. Proceedings SSST'93. 7-9 March 1993, IEEE Catalog Number 93TH0530-6, pp.124- 128.
2. E.W Palmer, and G.F.Ledwich, "Three Phase harmonic Modelling of Power System Loads", IEE Proceedings-C, Vol.140, No.3, pp.206-212, 1993.
3. R. Mancini, et-al, "An Area Substation Load Model in The presence of Harmonics", IEEE Transactions on Power Delivery, Vol.11, No.4, pp.2013-2019, 1996.
4. R. Lamedica, et-al, "A Model of Large Load areas for Harmonic Studies in Distribution Networks", IEEE Transactions on Power Delivery, Vol.12, No.1, pp.418-425, 1997.
5. S.. A. Soliman, A. M. Al-Kandari, and M E. El-Hawary, " Time Domain Estimation Techniques for Harmonic Load Models" Electric machines and Power Systems, Vol.25, pp.885-896, 1997.
6. M. Fauri, "Harmonic Modeling of Nonlinear Load by Means of Crossed Frequency Admittance Matrix" IEEE Transactions on Power Systems, Vol.12, No.4, pp. 1632-1638, 1997.
7. S. A. Soliman, N. H. Abbasy, and M. E. El-Hawary, " Frequency Domain Modeling and Identification of Nonlinear Loads Using a Least Error Squares Algorithm", Electric Power Systems Research. Vol.40, pp.1-6, 1997.

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8. S. Arnborg, et-al, "On Influence of Load Modeling for Under Voltage Load Shedding Studies", IEEE transactions on Power Systems, Vol.13, No.2, pp.395-400, 1998.
9. M. M. El-Arinin, "A Time Domain Load Modeling Technique and Harmonics Analysis", 8th International Conference on Harmonics and Quality of Power. Proceedings of 1998, Vol.2, pp.930-938, IEEE Catalog # 98EX227, 1998.
10. L. Sainz, et-al, "Load Modeling for Unbalanced Harmonic Power Flow Studies", 8th International Conference on Harmonics and Quality of Power. Proceedings of 1998, Vol.2, pp.665-671, IEEE Catalog # 98EX227, 1998.
11. Ahmed M. AL-Kandari, Khaled M. EL-Naggar," Recursive identification of harmonic loads in power systems". International Journal of Electrical Power & Energy Systems Volume 28, Issue 8, October 2006, Pages 531-536